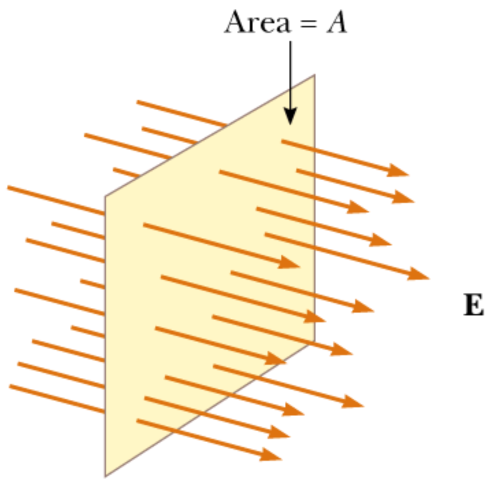
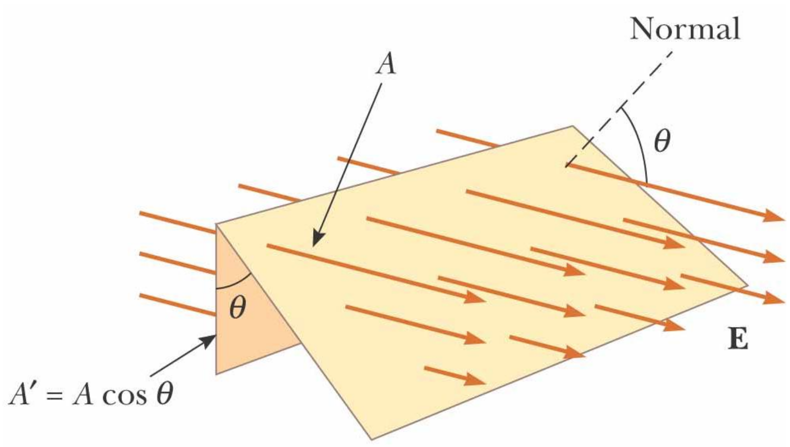
Gauss' Law

- The amount of the electric field – Electric Flux
- Simplifying field calculations – Gauss' Law
- Applications of Gauss' Law
- Conductors in equilibrium

Electric Flux





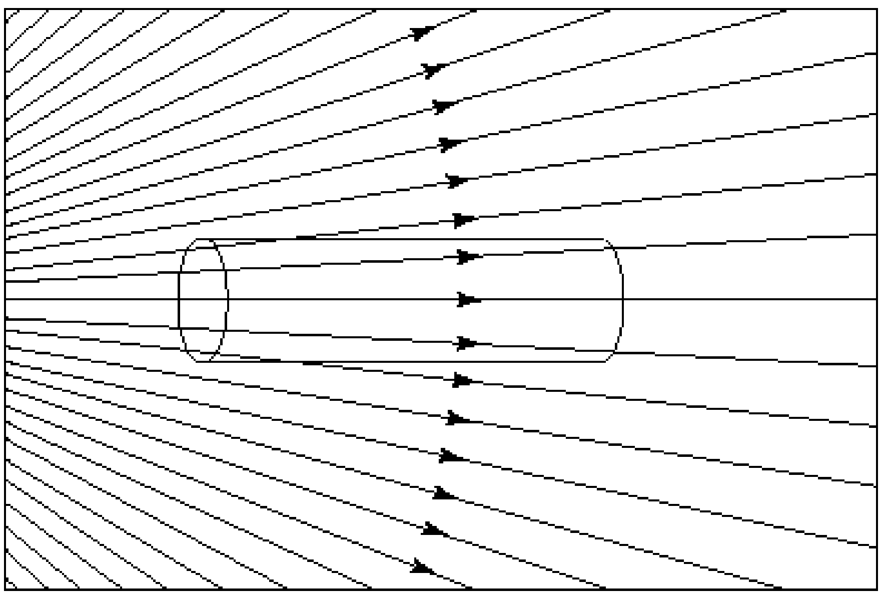
Electric flux is proportional to the total number of electric field lines through a surface

$$\Phi_E = EA \quad (N.m^2/C)$$

$$\Phi_E = \Phi'_E = EA' = EA\cos\theta$$

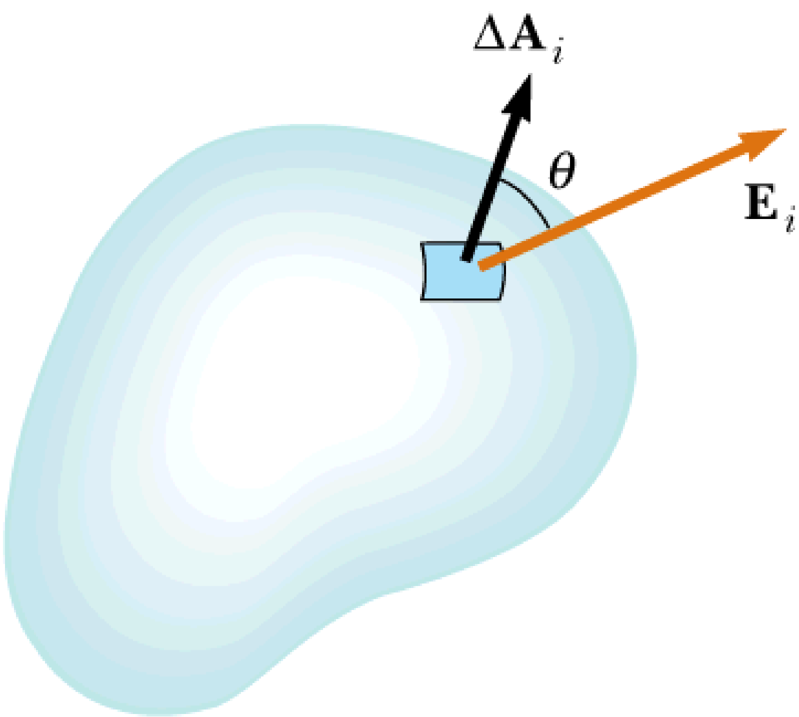
Concept Question

A cylindrical piece of insulating material is placed in an external electric field, as shown. The net electric flux passing through the surface of the cylinder is



1. positive.
2. negative.
3. zero.

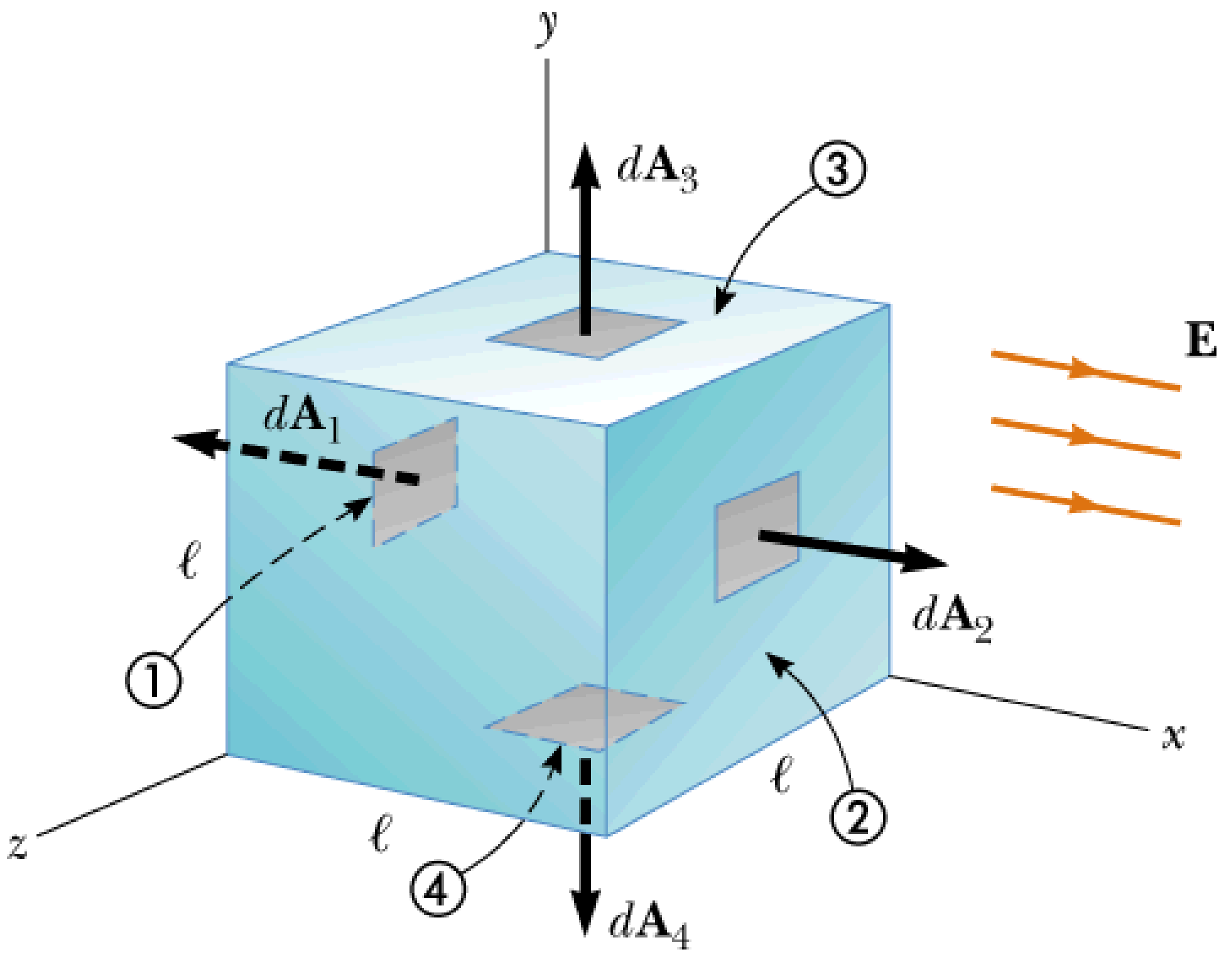
General Flux Definition



$$\Delta\Phi_E = E_i \Delta A_i \cos\theta = \vec{\mathbf{E}}_\mathbf{i} . \Delta\vec{\mathbf{A}}_\mathbf{i}$$

$$\Phi_E = \int_{surface} \vec{\mathbf{E}} . d\vec{\mathbf{A}}$$

Flux Through a Cube

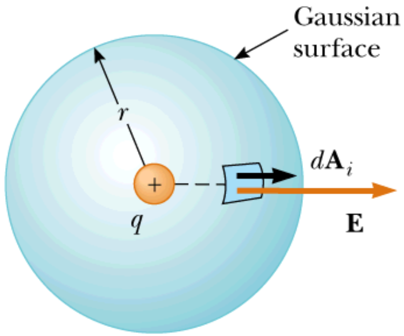


Gauss's Law

Start with a single charge and a spherical surface around it

Calculate the flux through the sphere

Generalize to any surface

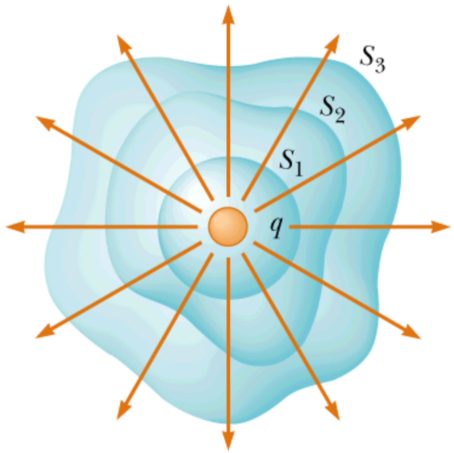


$$\Phi_E = \oint \vec{\mathbf{E}}.\mathbf{dA} = \oint EdA$$

$$\Phi_E = E\oint dA$$

$$\Phi_E = k_e \frac{q}{r^2}(4\pi r^2)$$

$$\Phi_E = \frac{q}{\varepsilon_0}$$



$$\Phi_E = \oint \vec{\mathbf{E}}.d\mathbf{A} = \frac{q_{in}}{\varepsilon_0}$$

Some Important Notes About Gauss' Law

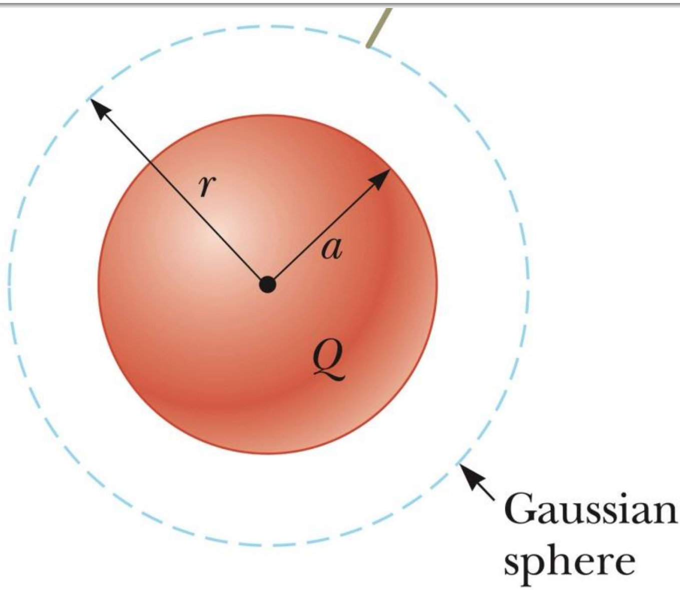
- A closed region that contains no charge has zero net flux through it.
- Even though the net flux is determined only by the charges inside the surface, the electric field at any given point on the surface is a result of all charges, inside and outside.
- Therefore, a zero flux through a closed surface does not imply a zero field at any point on the surface.

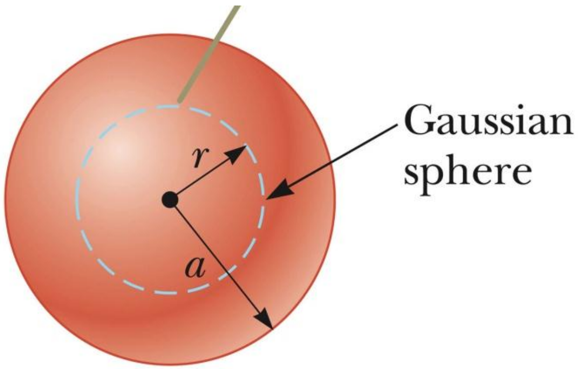
Application to Charge Distributions

Choosing a Gaussian Surface:

- The field over the surface is constant through symmetry.
- The field, **E** and the surface vector, *d***A** are parallel simplifying the dot product to an algebraic product.
- … or they are perpendicular, making the dot product zero.
- The field is zero over the surface.

Spherically Symmetric Charge Distribution



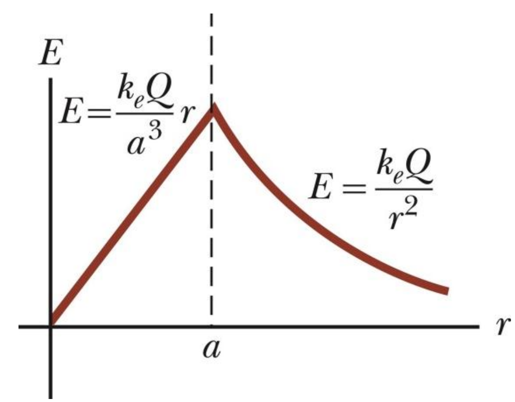


$$E = k_e \frac{Q}{r^2}$$

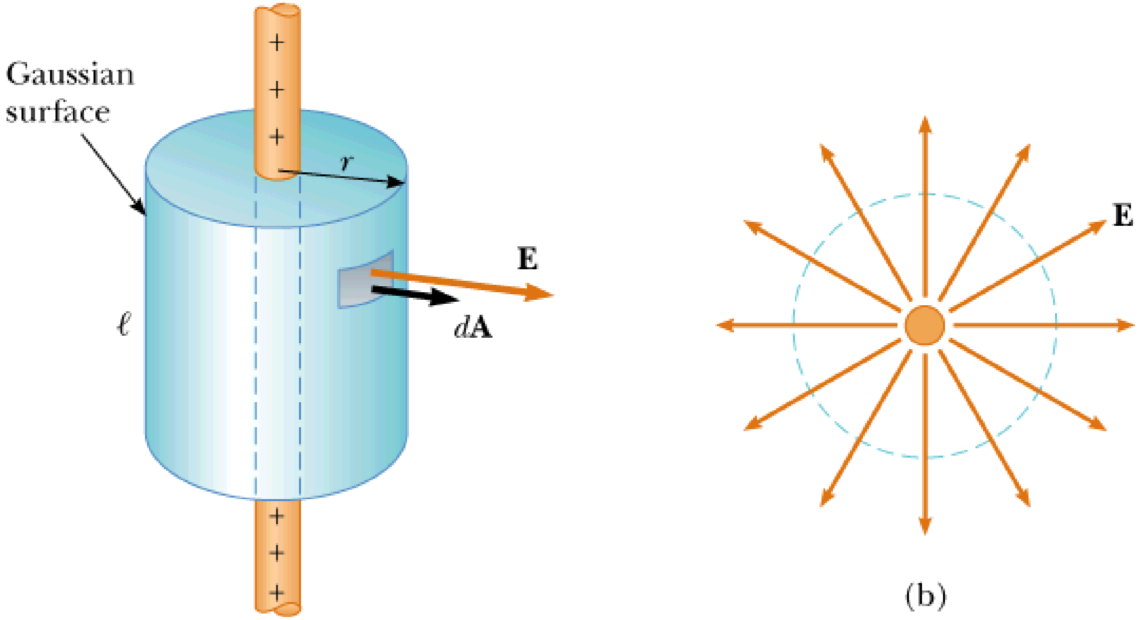
$$r > a$$

$$E = k_e \left(\frac{Q}{a^3} \right) r$$

$$r < a$$



Line of Charge – Cylindrical Symmetry



(a)

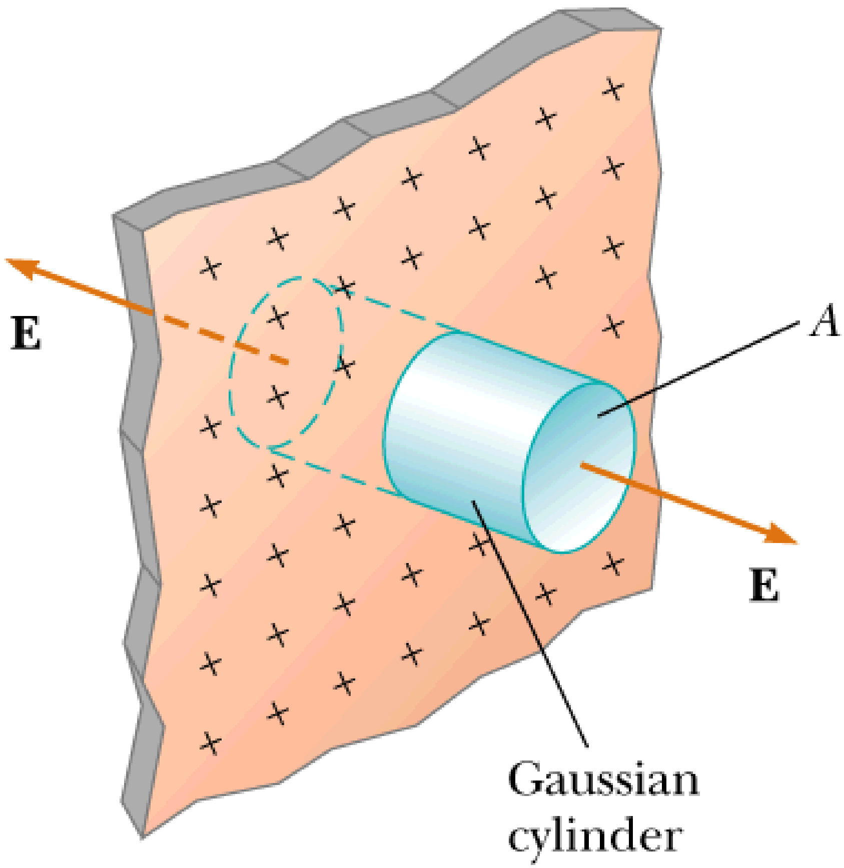
$$\Phi = \Phi_{side} + \Phi_{top} + \Phi_{bottom}$$

$$\Phi_{top} = \Phi_{bottom} = EA\cos 90^\circ = 0$$

$$\Phi = \Phi_{side} = EA_{side}\cos 0^\circ = E(2\pi r l)$$

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

Insulating Plane of Charge



$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}$$

$$\varepsilon_0 \left(EA + EA \right) = \sigma A$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

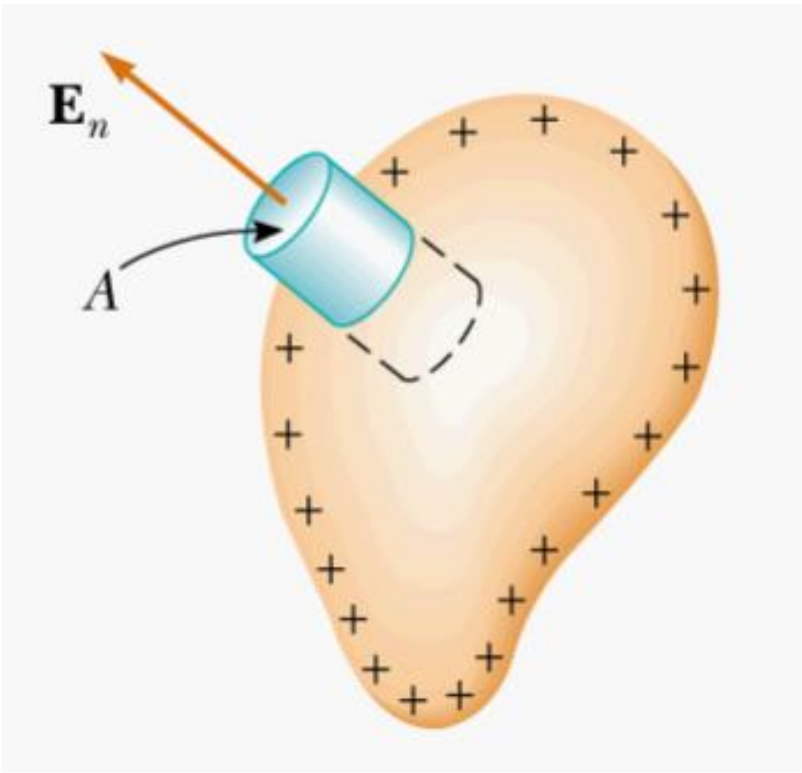
Conductors in Equilibrium

- Put a conductor in an electric field.
- The free electrons inside the conductor will accelerate in the opposite direction of the field lines.
- As the negative and positive charges separate, an internal field opposing the external field will be established.
- The acceleration of charges will continue until the internal field cancels out the external field and the conductor will reach electrostatic equilibrium.
- The electric field is zero everywhere inside a conductor at electrostatic equilibrium.

Conductors and Gauss's Law

- Take a Gaussian surface inside a conductor that is arbitrarily close to the surface.
- Since the electric field inside the conductor (and hence on the Gaussian surface) is zero, there is no net charge inside the surface.
- Since the surface is arbitrary, and can be made infinitesimally close to the outer surface of the conductor, any net charge on a conductor will reside on the surface.

Conductors and Gauss's Law



- The flux through the top surface is *EA*, since E is perpendicular to A (electrostatic equilibrium).
- Therefore the flux is zero through the side wall outside the conductor.
- The field, and hence the flux, through the surfaces inside the conductor are also zero.

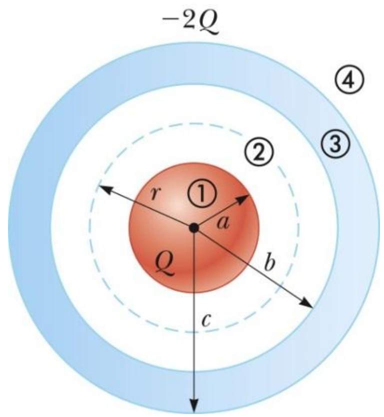
$$\Phi_E = \oint E dA = EA = \frac{q_{in}}{\varepsilon_0}$$

$$\boxed{E = \frac{q_{in}}{A\varepsilon_0} = \frac{\sigma}{\varepsilon_0}}$$

Conductors and Gauss' Law

- The electric field is zero everywhere inside a conductor at electrostatic equilibrium.
- Any net charge on a conductor will reside on the surface.
- The electric field just outside a conductor is perpendicular to the surface and is proportional to the charge density.
- The charge density is highest near parts of the conductor with the smallest radius of curvature.

A Sphere Inside a Spherical Shell



Region	Insulating Sphere	Conducting Sphere
$r < a$	$E = k_e \frac{Q}{a^3} r$	$E = 0$
$a < r < b$	$E = k_e \frac{Q}{r^2}$	$E = k_e \frac{Q}{r^2}$
$b < r < c$	$E = 0$	$E = 0$
$r > c$	$E = -k_e \frac{Q}{r^2}$	$E = -k_e \frac{Q}{r^2}$

Summary

- Flux through any closed surface is proportional to the net charge enclosed by the surface.
- Use symmetry to simplify calculations.
- All excess charge on a conductor will reside at the outer surface. The field inside the conductor is zero.

For Next Class

- Reading Assignment
  - Chapter 25 – Electric Potential
- WebAssign: Assignment 2